

# Universal scaling of the Hall resistivity in $\text{MgB}_2$ superconductors

W. N. Kang\*, Hyeong-Jin Kim, Eun-Mi Choi, Hun Jung Kim, Kijoon H. P. Kim, and Sung-Ik Lee

*National Creative Research Initiative Center for Superconductivity and Department of Physics, Pohang University of Science and Technology, Pohang 790-784, Republic of Korea*

The mixed-state Hall resistivity  $\rho_{xy}$  and the longitudinal resistivity  $\rho_{xx}$  in superconducting  $\text{MgB}_2$  thin films have been investigated as a function of the magnetic field over a wide range of current densities from  $10^2$  to  $10^4$  A/cm<sup>2</sup>. We observe a universal Hall scaling behavior with a constant exponent  $\beta$  of  $2.0 \pm 0.1$  in  $\rho_{xy} = A\rho_{xx}^\beta$ , which is independent of the magnetic field, the temperature, and the current density. This result can be interpreted well within the context of recent theories.

When a type-II superconductor is cooled down from a normal state to a superconducting state, the Hall effect shows very unusual features, which have been a long-standing problem and have remained an unresolved issue for more than three decades. The sign reversal of the Hall effect below  $T_c$  is one of the most interesting phenomena in the flux dynamics for high- $T_c$  superconductors (HTS) and has attracted both experimental and theoretical interest. It is now mostly accepted that the mixed-state Hall effect in type-II superconductors is determined by two contributions, quasiparticle and hydrodynamic. The sign of the quasiparticle term remains the same as that in normal state, but the hydrodynamic term can be negative in the mixed state, so sign reversal can take place [1–4]. Furthermore, a scaling behavior between  $\rho_{xy}$  and  $\rho_{xx}$  has been found in most HTS [5–12]. The puzzling scaling relation  $\rho_{xy} = A\rho_{xx}^\beta$  with  $\beta \sim 2$  has been observed for  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  crystals [5] and  $\text{Tl}_2\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$  films [6]. Other similar studies have reported  $\beta = 1.5 \sim 2.0$  for  $\text{YBa}_2\text{Cu}_3\text{O}_7$  (YBCO) films [7], YBCO crystals [8,9], and  $\text{HgBa}_2\text{CaCu}_2\text{O}_6$  films [10]. Even  $\beta \sim 1$  was reported for heavy-ion-irradiated  $\text{HgBa}_2\text{CaCu}_2\text{O}_6$  thin films [11].

A number of theories have been proposed to explain the scaling behavior between  $\rho_{xy}$  and  $\rho_{xx}$ . The first theoretical attempt was presented by Dorsey and Fisher [13]. They showed that near the vortex-glass transition,  $\rho_{xy}$  and  $\rho_{xx}$  could be scaled with an exponent  $\beta = 1.7$ , and they explained the experimental results of Luo *et al.* for YBCO films [7]. A phenomenological model was put forward by Vinokur *et al.* [14]. They claimed that in the flux-flow region,  $\beta$  should be 2 and independent of the pinning strength. Their result was consistent with the observed exponent in Bi-, Tl- and Hg-based superconductors only for the Hall data measured in high magnetic fields [5,6,12]. Another phenomenological model was proposed by Wang *et al.* [15], who claimed that  $\beta$  could change from 2 to 1.5 as the pinning strength increased, which agreed with the results reported for YBCO crystals [8,9] and Hg-1212 films [10].

The Hall scaling behavior, therefore, is a complicated phenomena, which seems strongly depend on the type of HTS. However, from the experimental Hall data reported in previous papers [5–12], one can find a general trend. At higher fields, the scaling range is wide and shows a

universal value of  $\beta \sim 2$ , which is independent of the field and the pinning strength. At lower fields, the scaling range is relatively narrow because the contribution from the hydrodynamic term is comparable to that of quasiparticle term; thus, the value of  $\beta$  does not appear to be constant. Moreover, no Hall scaling has been reported in the field-sweep data, which provides decisive proof for the temperature dependence of the Hall scaling behavior.

The  $\text{MgB}_2$  superconductor is a very interesting sample for investigating the flux dynamics. Different from HTS,  $\text{MgB}_2$  shows no Hall sign anomaly in the mixed state and has a rather simple vortex phase diagram [16]. The absence of sign anomaly implies that the hydrodynamic contribution is very small or negligible. Thus, the  $\text{MgB}_2$  compound is probably the best candidate for probing whether the Hall scaling is universal or not because we need only consider the quasiparticle term of the Hall conductivity, which is consistent with the universal Hall scaling theory [14].

In this Letter, we report the first demonstration of a universal scaling behavior of the Hall resistivity in  $\text{MgB}_2$  superconducting thin films, and the results can be well described using recent theories. We confirmed that the scaling exponent  $\beta \sim 2$  is universal, which is independent of the temperature, the magnetic field, and the pinning strength. In order to test the pinning strength dependence of the scaling behavior, we measured the Hall effect for two orders of magnitude of the current density. Based on our results, we will show that the universal Hall scaling law is also valid for HTS in high fields where the hydrodynamic contribution in the Hall effect is very small compared to the quasiparticle contribution.

The  $\text{MgB}_2$  thin films were grown on  $\text{Al}_2\text{O}_3$  (1  $\bar{1}$  0 2) substrates under a high-vacuum condition of  $\sim 10^{-7}$  Torr by using pulsed laser deposition and postannealing techniques. The fabrication process and the normal-state transport properties of  $\text{MgB}_2$  thin films are described in detail elsewhere [16,17]. The X-ray diffraction patterns indicated highly c-axis-oriented thin films perpendicular to the substrate surface and with a sample purity in excess of 99%. The critical current density at 15 K and under a self-field condition was observed to be on the order of  $10^7$  A/cm<sup>2</sup> [18]. Standard photolithographic

techniques were used to produce thin-film Hall bar patterns, which consisted of a rectangular strip (1 mm  $\times$  3 mm) of MgB<sub>2</sub> film with three pairs of sidearms. The narrow sidearm width of 0.1 mm was patterned so that the sidearms would have an insignificant effect on the equipotential. Using this 6-probe configuration, we were able to measure simultaneously the  $\rho_{xx}$  and the  $\rho_{xy}$  at the same temperature. To achieve good ohmic contacts, we coated Au film on the contact pads after cleaning the sample surface by using Ar-ion milling. Fine temperature control was crucial since the Hall signal of MgB<sub>2</sub> is very small. The magnetic field was applied perpendicular to the sample surface by using a superconducting magnet system, and the applied current densities were  $10^2$ – $10^4$  A/cm<sup>2</sup>. The Hall voltage was found to be linear in both the current and the magnetic field.

Figure 1(a) shows the temperature dependence of  $\rho_{xx}$  for MgB<sub>2</sub> thin films for various magnetic fields up to 5 T and for applied current densities of  $10^3$  and  $10^4$  A/cm<sup>2</sup>. At zero field, the onset transition temperature ( $T_c$ ) was 39.2 K and had a narrow transition width of  $\sim 0.15$  K, as judged from the 10 to 90% superconducting transition. As current density was decreased, the large enhancement of  $T_c$  was clearly observed. This is similar pinning effect being enhanced by introducing columnar defects in HTS [6,8,11]. This result indicates that we can investigate the pinning strength dependence of the Hall scaling behavior by changing the applied current density.

The corresponding  $\rho_{xy}$  is plotted in Fig. 1(b). The same trend as seen in the  $\rho_{xx} - T$  curves was observed with increasing current density. No sign change was detected for magnetic fields up to 5 T over a wide range of current densities from  $10^3$  to  $10^4$  A/cm<sup>2</sup>. This result implies that in the MgB<sub>2</sub> superconductor, the hydrodynamic contribution for the mixed-state Hall conductivity is very small or negligible compared to the quasiparticle contribution. Thus, this compound is probably the most suitable sample to prove the Hall scaling behavior. The overall feature of the temperature dependence is quite different from that of HTS [5,6,8,10–12], in which a dip structure is observed near  $T_c$  due to the negative contribution by the hydrodynamic term. The normal-state  $\rho_{xy}$  at 5 T decreases linearly with increasing temperature from 30 to 40 K, which is consistent with the temperature dependence of  $\rho_{xy}$  above  $T_c$  [17].

Figure 2 shows the field dependence of (a)  $\rho_{xx}$  and (b)  $\rho_{xy}$  for MgB<sub>2</sub> films at various temperatures from 28 to 40 K and current densities of  $10^3$  and  $10^4$  A/cm<sup>2</sup>. A very small and positive magnetoresistance was observed at 40 K and 5 T. With decreasing temperature and current density, the superconducting transitions of  $\rho_{xx}$  and  $\rho_{xy}$  became broad, showing that a relatively wide vortex-liquid phase had been formed. Similar to the  $\rho_{xx} - T$  behavior, large increases of zero-resistance transition were observed in both the  $\rho_{xx}$  and the  $\rho_{xy}$  data when a lower current density was applied, which indicates that the pin-

ning force can be adjusted systematically by changing the applied current density. As the magnetic field was increased,  $\rho_{xy}$  grew gradually without changing its sign up to an upper critical field (marked by the arrow), which is different from the previous observation [19] in polycrystalline MgB<sub>2</sub>. In the normal state above upper critical fields, we found a quite linear  $H$ -dependence of  $\rho_{xy}$ . This is clearly different from the HTS case and suggests a relatively simple flux dynamics in MgB<sub>2</sub>.

The scaling behavior of  $\rho_{xy}$  between  $\rho_{xx}$  for the temperature-sweep data (Fig. 1) of MgB<sub>2</sub> films is plotted in Fig. 3 for various fields from 1 to 5 T and over a wide range of current densities from  $10^2$  to  $10^4$  A/cm<sup>2</sup>. The universal Hall scaling with an exponent of  $2.0 \pm 0.1$  is evident and is independent of the magnetic field and the current density. More strikingly, this universal scaling generally occurs in the regimes of the flux flow, such as the free-flux-flow, the thermally activated flux-flow, and the vortex-glass regions [20]. This result is different from those for HTS where the scaling relation is valid only in the thermally activated flux-flow and vortex-glass regions [5–12].

Other supporting data for the universal Hall scaling of the field-sweep data (Fig. 2) can be seen in Fig. 4. We find a universal value of  $\beta = 2.0 \pm 0.1$ , which does not depend on the temperature or on the current density at (a)  $10^4$  and (b)  $10^3$  A/cm<sup>2</sup>. This is the first observation using the field-sweep measurements and provides decisive evidence for the temperature independence of the Hall scaling. Thus, we have confirmed that the flux-flow  $\rho_{xy}$  is determined by  $\rho_{xx}$  and that the relation is independent of the magnetic field, the temperature, and the current density. This universal behavior of the Hall scaling is our principal finding, and this observation will have serious implications for the physics of mixed-state Hall behavior, as discussed below.

Vinokur *et al.* [14] proposed a universal Hall scaling theory based on a force balance equation where the Lorentz force  $\mathbf{f}_L$  acting on a vortex is balanced by the usual frictional force  $-\eta\mathbf{v}$  and the Hall force  $\alpha\mathbf{v} \times \mathbf{n}$  with  $\mathbf{v}$  being the average velocity of vortices and  $\mathbf{n}$  being the unit vector along the vortex lines. The coefficient  $\alpha$  is related to the Hall angle by means of  $\tan\Theta_H = \alpha/\eta$ . If a pinning force of  $-\gamma\mathbf{v}$ , which renormalizes only the frictional term but does not affect the Hall force term, is included, the equation of vortex motion is given by

$$(\gamma + \eta)\mathbf{v} + \alpha\mathbf{v} \times \mathbf{n} = \mathbf{f}_L. \quad (1)$$

Using this force balance equation, we can easily calculate the relation between  $\rho_{xy}$  and  $\rho_{xx}$ :

$$\rho_{xy} = A\rho_{xx}^2, \quad (2)$$

where  $A = \alpha/(\Phi_0 B)$  with  $\Phi_0$  being the flux quantum. Equation (2) gives a universal scaling law with  $\beta = 2$ ,

which is independent of the magnetic field, the temperature, and the pinning strength. Furthermore, this relation can be applied to the entire flux-flow region. Our experimental data in Figs. (3) and (4) can be interpreted completely by this simple theory. This excellent consistency between theoretical and experimental results is very important in the field of vortex dynamics since we are able to set up the basic equation of vortex motion, Eq. (1), which should provide a significant direction for future investigations on vortex dynamics. If this theory is to be generally accepted, however, experimental observation of HTS with  $\beta$  ranging from 1 to 2 should be explained.

Now, we discuss the spread value of  $\beta$  ( $1 < \beta < 2$ ) reported for HTS. An interesting microscopic approach based on the time-dependent Ginzburg-Landau theory has been proposed in a number of papers [1–3]. According to this model, the mixed-state Hall conductivity  $\sigma_{xy}$  in type II superconductors is determined by the quasiparticle contribution  $\sigma_{xy}^{(q)}$  and the hydrodynamic contribution  $\sigma_{xy}^{(h)}$  of the vortex cores,  $\sigma_{xy} = \sigma_{xy}^{(q)} + \sigma_{xy}^{(h)}$ . Since  $\sigma_{xy}^{(h)}$  is determined by the energy derivative of the density of states [2,3], if that term is negative and dominates over  $\sigma_{xy}^{(q)}$ , a sign anomaly can appear. This theory is consistent with experimental data for HTS [4]. This microscopic theory suggests that Hall scaling can be broken in the case where  $\sigma_{xy}^{(h)}$  is comparable to  $\sigma_{xy}^{(q)}$  because those terms have opposite signs. Indeed,  $\beta$  was observed to be less than 2 at low fields whereas a universal value of  $\beta \sim 2$  was observed at higher fields where  $\sigma_{xy}^{(h)}$  is very small compared to  $\sigma_{xy}^{(q)}$  (for example,  $\beta \sim 2$  was observed in Hg- and Tl-based superconductors for  $H = 9 - 18$  T [12]). For the  $\text{MgB}_2$  compound, since no sign anomaly was detected, we can say with confidence that  $\sigma_{xy}^{(h)}$  is very small or negligible; thus, universal Hall scaling holds over a wide range of fields. One can conclude from these results, that Hall scaling is universal under conditions where the  $\sigma_{xy}^{(q)}$  term dominates the  $\sigma_{xy}^{(h)}$  term; thus we can then explain all the reported data related Hall scaling issues for HTS [5–12].

In summary, we have found a universal Hall scaling behavior between  $\rho_{xy}$  and  $\rho_{xx}$  in  $\text{MgB}_2$  thin films, which is in good agreement with the high-field Hall data from Bi-, Hg- and Tl-based HTS [5,12]. Our Hall data can be completely explained within the context of the universal Hall scaling theory. We also show that  $\rho_{xy}$  can scale as  $\rho_{xx}^2$  in cases where the  $\sigma_{xy}^{(q)}$  term dominates  $\sigma_{xy}^{(h)}$  term. With a simple phenomenological theory [14] and a microscopic theory [1–3], we are able to explain the Hall scaling behavior in HTS, which has been debated for a long time. We believe that these results will provide new insight into the future theoretical studies on the vortex dynamics of superconductivity.

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\* E-mail address: wn Kang@postech.ac.kr

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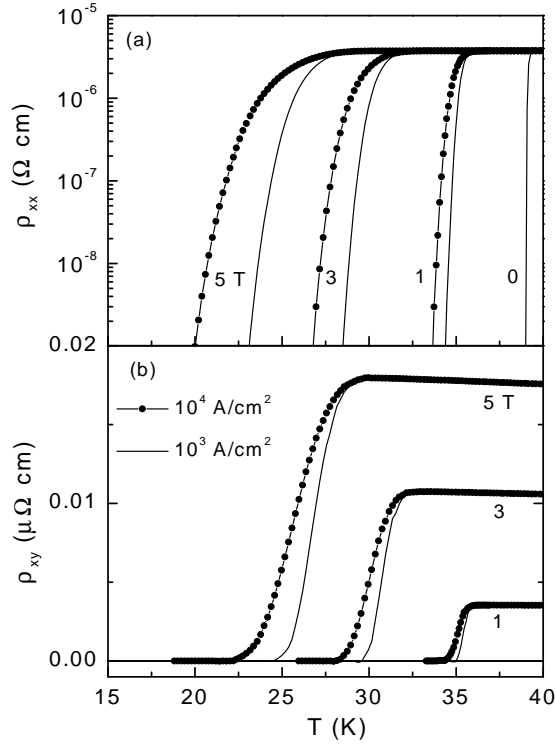


FIG. 1. Temperature dependences of (a)  $\rho_{xx}$  and (b)  $\rho_{xy}$  for MgB<sub>2</sub> thin films in a magnetic field up to 5 T and at current densities of  $10^3$  and  $10^4$  A/cm<sup>2</sup>.  
Fig. 1. Kang et al.

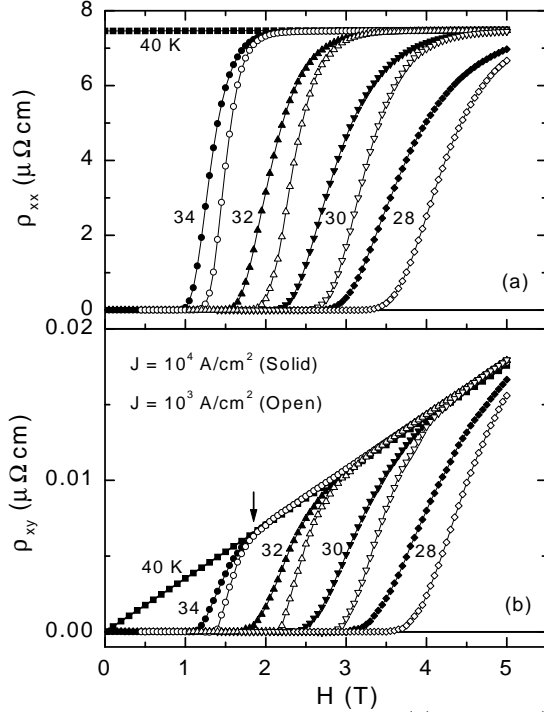


FIG. 2. Magnetic field dependences of (a)  $\rho_{xx}$  and (b)  $\rho_{xy}$  curves for MgB<sub>2</sub> thin films at temperatures from 28 to 40 K and current densities of  $10^3$  and  $10^4$  A/cm<sup>2</sup>.  
Fig. 2. Kang et al.

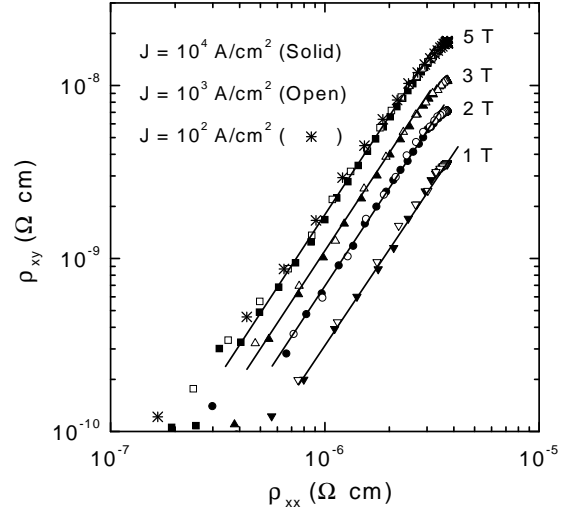


FIG. 3. Scaling behaviors between  $\rho_{xy}$  and  $\rho_{xx}$  for the temperature-sweep data of MgB<sub>2</sub> thin films in magnetic fields from 1 to 5 T and at current densities of  $10^2$ ,  $10^3$ , and  $10^4$  A/cm<sup>2</sup>. The universal Hall scaling with an exponent of  $2.0 \pm 0.1$ , which is independent of magnetic fields and current densities, is evident.  
Fig. 3. Kang et al.

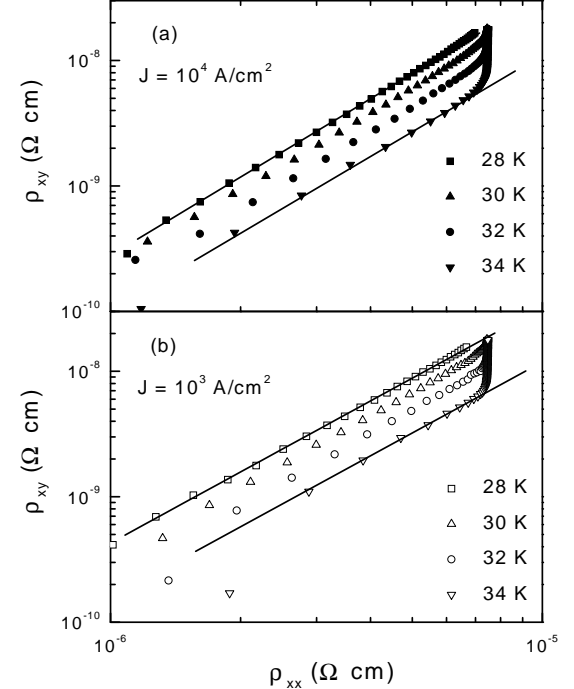


FIG. 4. Scaling behaviors between  $\rho_{xy}$  and  $\rho_{xx}$  for the field-sweep data of MgB<sub>2</sub> thin films measured at temperatures from 28 to 34 K and current densities of (a)  $10^4$  and (b)  $10^3$  A/cm<sup>2</sup>. A universal Hall scaling with an exponent of  $2.0 \pm 0.1$ , which is independent of the temperature and the current density, is clearly seen.  
Fig. 4. Kang et al.